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The Numerical Determining of the Magnetic Dynamo Growth Rate During a Parametric Resonance

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Abstract.

Classical parametric resonance in oscillatory systems can be characterized by the appearance of exponential growth near the selected frequencies of parametric excitation. However, if the oscillatory system, even without parametric excitation, has exponential growth (which is typical, for example, for dynamo-systems describing stellar magnetic cycles), then the appearance of parametric excitation can lead not only to the change in exponential growth, but also to a splitting of oscillation frequencies, and, accordingly, to the appearance of unharmonic oscillations. The numerical analysis of such a structure with an exponentially growing amplitude is a complex problem for which we propose a special approach. Using the proposed approach, we analyze the dependence of generation rates and frequencies on the excitation frequency for a well-known Parker-type dynamo-system.

Keywords: MHD dynamo; parametric resonance; numerical methods.

Introduction.

Many astrophysical dynamo systems have their own oscillation frequencies, so for such systems the question of the periodic impact of their cosmic "neighbors" (satellites, planets and stars) becomes topical. The answer to this question can be sought both analytically and numerically [Serenkova et al., 2023], but it should be taken into account that the analysis must be highly accurate, since, for example, the effects of classical parametric resonance appear in the immediate vicinity of doubled and multiple frequencies. Here we use numerical analysis and develop a method to find dynamo growth rates and frequencies for a simple well-known dynamo model in a spherical thin convective shell. This model was proposed by Eugene Parker in 1955 to describe solar magnetic activity and is a direct consequence of averaging the magnetic induction equation over a random velocity field [Steenbeck et al., 1966], while the field itself, for convenience, is decomposed into a poloidal and toroidal component (A and B, respectively) with azimuthal spherical symmetry [Parker, 1955]:

$$\begin{aligned} \dot{A} &= R_\alpha B + A_{\theta\theta} - \mu^2 A, \\ \dot{B} &= R_\omega (A\sin(\theta))_\theta + B_{\theta\theta} - \mu^2 B, \end{aligned} \tag{1}$$

here the parameter R_α is responsible for hydrodynamic helicity, R_ω is responsible for differential rotation, the product $R_\alpha * R_\omega$ gives the dynamo number, and μ determines the radial part of the diffusion. The subscript represents the derivative of the zenith angle.

Hypothetically, some periodic influence on the system (for example, binary stars on each other or exoplanets on their star) can lead to periodic changes in R_α or R_ω, which, in turn, can cause the parametric resonance, since the magnetic field also oscillates with time. To analyze this situation, the following change is added to the system:

$$R_\omega \to R_\omega (1 + \sigma \sin(\omega t)), \tag{2}$$

where σ is the amplitude of the parametric effect.

The Parker system can be simplified by reducing it by a low-mode approximation to a system of ordinary differential equations, e.g., [Kalinin and Sokoloff, 2019], [Tarbeeva et al., 2011], [Kalinin and Sokoloff, 2018]. The analysis of the low-mode system is given in the work [Serenkova et al., 2023], where a special kind of resonant curve of the dependence of the exponential growth rate on the excitation frequency is shown. At the same time, for each frequency of parametric excitation, the velocity was determined as follows: the maxima of the oscillations were found, which were supposed to form a growing exponent, natural logarithms were taken from the obtained values and approximated by a straight line (Figure 1, left panel) using the least squares method [Stigler, 1986]. However, this method turned out to be inapplicable for the analysis of oscillations obtained by numerically solving a system of partial differential equations, due to the strong noise of the resonant pattern. The main problem is related to a more complex pattern of oscillations than with harmonic oscillations with exponentially increasing amplitude. In particular, there is a splitting of frequencies, so the local exponential growth of the maxima can be replaced by a decrease. Due to this reason, the histogram of the slope angles between the logarithms of neighboring maxima (Figure 1, right panel) has the form not of a single peak, but of some limited corridor of values, within which the true value of the growth rate is to be found.

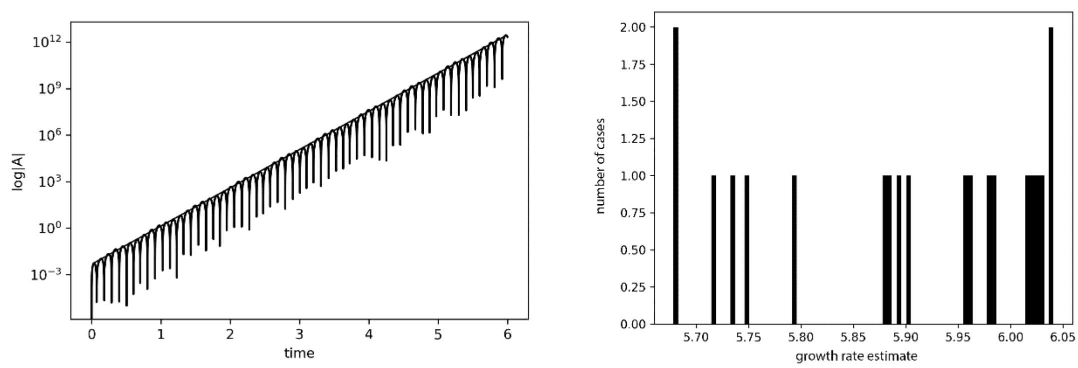

Figure 1. *Left panel: exponential growth of field A at zenith angle $\theta = \pi/4$. Right panel: histogram of slope angles between logarithms of neighboring maxima with exponential growth of the magnetic field in the Parker system.*

A new method for determining the speed.

In order to determine the actual rate of exponential growth, the following technique is used: for the already obtained solution of the system in the absence of diffusion for a given angle $\theta = \pi/4$, a histogram of the slope angles between the logarithms of the adjacent two maxima is constructed (Figure 1, right panel). The smaller slope is denoted by $\gamma_{\min}$, the larger one, respectively, $\gamma_{\max}$, and then the segment between them is divided into 1000 parts of length $d\gamma = (\gamma_{\max} - \gamma_{\min})/1000$. The diffusionless solution is multiplied by $e^{-\gamma t}$, iterating γ from $\gamma_{\min}$ to $\gamma_{\max}$ in increments of $d\gamma$ until the solution stops growing, this value is γ and is declared the exponential growth rate of the system by this frequency. For the numerical algorithm, the stopping criterion will be the location of the largest maximum of oscillations (not counting the first few pieces when the system has not yet entered its mode) in the left half of the graph, that is, if the time is counted in the program up to $T = 6$, then the greatest maximum is expected to be seen up to $t = T/2 = 3$. An example of oscillations with such a γ is shown in (Figure 2, left panel).

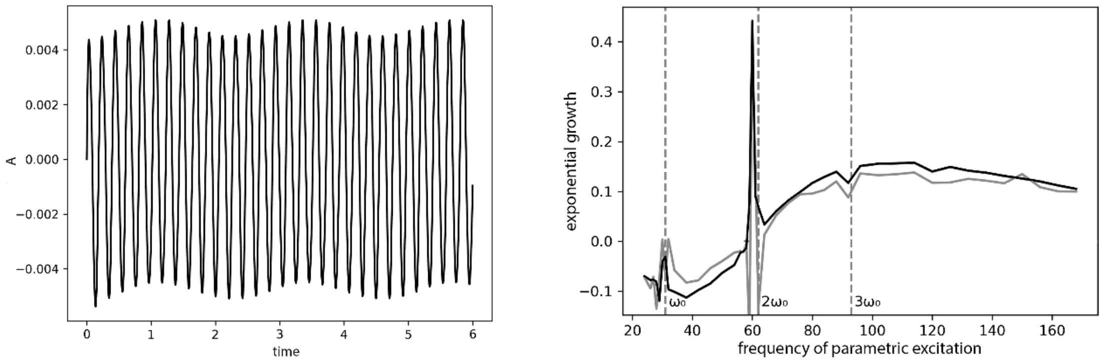

Figure 2. *Left panel: a type of oscillation for a solution with suppressed generation at a sufficiently accurately calculated exponential growth rate at zenith angle $\theta = \pi/4$. Right panel: the dependence of the exponential growth rate on the frequency of parametric excitation in the Parker system during calculations in the old way (gray) and new (black), vertical lines mark the natural, doubled and tripled frequencies of the system.*

The advantage of the new method for numerically determining the exponential growth rate of a system is that, in addition to a significant increase in accuracy and, as a result, a smoother dependence of this rate on the excitation frequency (Fig. 2, right panel), it becomes possible to do without any methods averaging values from the histogram. This averaging, which was included, in particular, in the least squares method used earlier, introduced the largest error due to the uniform distribution of growth rate estimates over individual maxima. In general, it would be possible to increase the number of maxima, but this requires more numerical power and results in less accuracy. Instead, the speeds calculated by the new method have a specific mathematical meaning - when multiplied by the corresponding exponent with a negative exponent, growth stops. The speed of calculation by the new method is also low, since for this the program only multiplies the ready-made solution by a number and looks for its local maximum.

Oscillation frequencies of the system.

Obtaining the correct exponential growth rate of a dynamo system with parametric action now allows us to more accurately determine the frequencies at which this system oscillates. By the type of vibrations (Figure 2, left panel), it can be understood that there should be several such frequencies, since there is some similarity of beats. The easiest way to determine the oscillation frequencies is using the Fourier transform [Fourier, 1822], but it is simply inhumane to force a computer to numerically reproduce such a transformation for an exponentially growing solution. With the advent of a sufficiently precisely found exponential growth rate, the solution can be made to stop growing by multiplying it by the corresponding exponent, and then calculate the spectrum and frequencies of oscillations. The Fourier spectra for different frequencies of parametric exposure are shown in Figure 3. On all graphs there are main peaks at the natural frequency of the system (its calculation is performed in the absence of parametric effects), as well as at least one other low peak, the position of which varies depending on the frequency of exposure (Figure 4), the remaining peaks are even lower and lie to the right of both previous ones. Figure 3 clearly shows how when the frequency of exposure reaches values in the vicinity of the doubled natural frequency of the system, the second peak merges with the first, forming beats. It is worth noting that this side frequency increases with the frequency of the disturbance, but does not coincide with it.

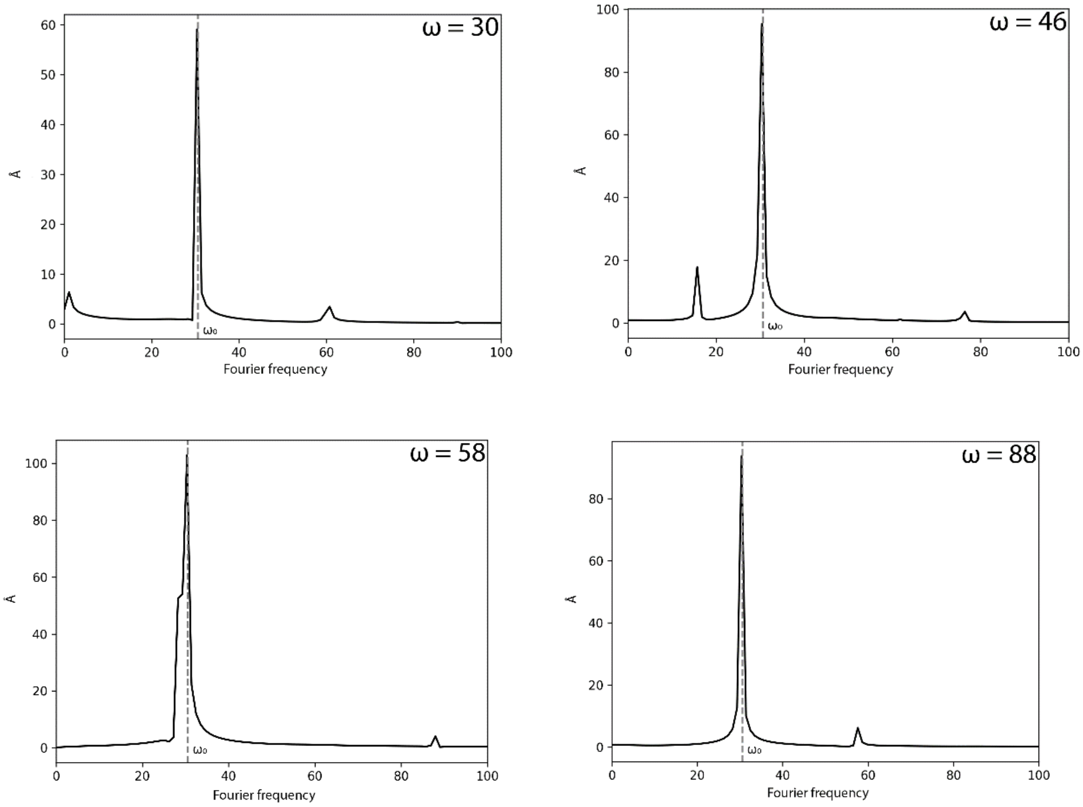

Figure 3. *The spectrum of the solution with suppressed generation for different frequencies of parametric excitation.*

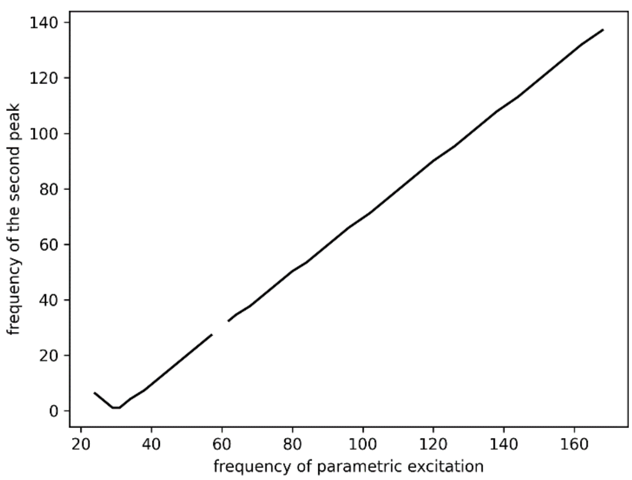

Figure 4. *The position of the second largest peak of the Fourier transform depending on the frequency of excitation. In the area of the doubled frequency of the system (about ω = 60), the peak merges with the main one.*

Conclusions.

The paper proposes a numerical algorithm that makes it possible to calculate the exponential growth rate of complexly oscillating solutions with any predetermined accuracy. The algorithm has demonstrated its advantage over previously used algorithms for harmonic oscillations with exponentially growing amplitude. Regarding the problem of parametric excitation in the Parker dynamo model, he made it possible to restore the dependence of the growth rate and the frequency Fourier spectrogram, which demonstrated fundamental differences from the classical parametric resonance. In particular, it was shown that, in addition to the generation rate peak at the doubled frequency, dynamo models can exhibit a resonant background (“non-selective resonance”), which corresponds to suppression of generation at low frequencies of the excitation force and enhancement of generation at high frequencies. In this case, the ratio of the amplitudes of the selective and non-selective resonance depends, as it turned out, on the parameters of the problem. Frequency Fourier spectrograms, which became possible to calculate after applying the developed algorithm to determine the generation rate, demonstrated the appearance of an additional peak along with the natural frequency of the system with a frequency that linearly depends on the frequency of the driving force. A distinctive feature in this case is that near the doubled (to the natural) frequency of the exciting force, a coincidence of frequencies is observed, and, accordingly, the behavior of the system takes the form of beats, which interferes with the operation of simpler mechanisms of numerical analysis.

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