

On Modified Yukawa Screening in Dense Hydrogen Plasma

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Abstract

The plasma of low and moderate non-ideality is of interest in solar and stellar plasma. The previous research of modeling a collective plasma influence onto the emitter in plasma was conducted with the modeling of electron screening influence by the usage of Debye screening mechanism. Although this was a known erroneous presumption a preliminary results has shown a expected behavior in respect to particles in plasma densities as well as temperatures that was present in modeled plasma. The more thorough research has shown that the modified Yukawa type screening is more applicable. Here are presented preliminary results of plasma influence onto the emitter potential in pure hydrogen plasma. This is the further step in development of more detailed analysis of collective phenomena in area of moderate to dense plasma influence.

Keywords: hydrogen dense plasma; Solar plasma; plasma microfield;

1. Theoretical remarks

In dense plasma, the inter-particle Coulomb interaction becomes dominant over the thermal kinetic energy [Fortov et al., 2006.]. Under such conditions, the coupled system of particles exhibits behavior reminiscent of a crystalline solid. For the case of hydrogen, a simplified form of the non-ideality parameter Γ and Wigner-Seitz radius are given by

$$\Gamma = \frac{E_p}{E_k} = \frac{e^2}{kT r_{WS}} \sim e^2 N_e^{1/3} \beta, \quad \beta = 1/kT, \quad \text{and} \quad r_{WS} = \left(\frac{3}{4\pi N_e} \right)^{1/3}. \quad (1)$$

At high Γ values ($\Gamma > 1$), the ions arrange themselves in a structured, nearly crystalline configuration to minimize Coulomb energy. In particular, the system tends toward a locally ordered state resembling a face-centered cubic (FCC) lattice, which is one of the most efficient packing arrangements for spheres in three dimensions. The FCC structure, found in many solid metals, maximizes the nearest-neighbor distance while minimizing the free energy of the system. Similarly, in strongly coupled plasmas, the ions adopt configurations where their mutual repulsion is optimized, leading to short-range order analogous to that of an FCC crystal.

Inside such a dense-packed, crystal-like structure, screening mechanisms become crucial in determining the effective local pseudo-potential. The electron cloud surrounding ions modifies the bare Coulomb interaction, leading to screened potentials such as the Yukawa or Debye-Hückel form, which significantly influence the stability and dynamics of the ion lattice. This screening effect must be accounted for when describing the thermodynamic and transport properties of strongly coupled plasmas.

The interaction in plasma has been successfully modeled using a cut-off Coulomb potential in previous studies, as demonstrated in [Dimitrijević et al., 2018.], [Ignjatović et al., 2009.], [Mihajlov et al., 2015.], and [Salpeter. 1954.]. While plasma effects are typically expected to dominate at distances far from the ionic core (compared to its characteristic radius), they can also significantly influence the form of ab initio-derived pseudo potentials.

For hydrogen, the range $0.1 \leq \Gamma \leq 1.5$ represents a particularly important regime where the system transitions from weakly coupled ($\Gamma < 1$) to moderately coupled ($\Gamma \geq 1$) plasma. In this interval:

Near $\Gamma \leq 0.1$, the system begins to show noticeable deviations from ideal plasma behavior.

At $\Gamma \geq 1$, the potential energy becomes comparable to the kinetic one (the definition of Γ).

Approaching $\Gamma \geq 1.5$, short-range order starts to emerge while maintaining fluid-like properties.

This transition region is crucial for understanding hydrogen plasmas in many astrophysical and laboratory conditions. For other ion species with different charge states and masses, the model’s behavior needs more comprehensive examination, as the relevant ranges and coupling effects may differ substantially.

Yukawa-like Screening in Coupled Plasmas

The screening behavior in plasmas undergoes significant changes across different coupling regimes:

Weak Coupling Regime

In the Debye-Hückel limit, the screening is characterized by the Debye length in SI as well as atomic units:

$$\lambda_D = \sqrt{\frac{k_B T}{4\pi n_e e^2}} = \sqrt{\frac{T}{4\pi n_e}}, \quad (2)$$

where the screened potential follows the classical Yukawa form.

Intermediate Coupling Regime ($0.3 < \Gamma \leq 3.0$)

As coupling increases, nonlinear effects become important. We implement a modified Yukawa screening where:

$$\lambda_{\text{eff}} = \frac{\lambda_D}{1 + \alpha \Gamma^{3/2}}, \alpha \approx 0.2, \quad (3)$$

with the corresponding screened potential:

$$V(r) = -\frac{Z}{r} e^{-r/\lambda_{\text{eff}}}. \quad (4)$$

Generalized Screening Approach

For arbitrary base potentials $V_{\text{base}}(r)$, the screening can be incorporated through:

$$V_{\text{screen}}(r) = V_{\text{base}}(r) e^{-r/\lambda_{\text{eff}}}, \quad (5)$$

where λ_{eff} may be determined self-consistently. This approach maintains consistency with the OCP (One-Component Plasma) limit while allowing for more complex interactions.

2. Results and Discussion

The screening behavior in moderately coupled plasmas exhibits characteristic features that depend on both the number of ionic layer shells considered in calculation, the system temperature, as well as the inter ionic distance related to Wiegner-Seitz radius r_{WS} from the Eq. (1). For the purpose of the simplified analysis on all figures a behavior of the ratio of the

calculated potential over a Coulomb potential, $V_{\text{avg}}(r)/V_{\text{Coul}}(r)$, is considered. Figures 1-4 demonstrate these effects. The number of shells indicate how many shells with ions on roughly another distance layer are taken into account. They roughly assemble a shell like structure, like onion shells around a emitter, central ion, considered into calculation.

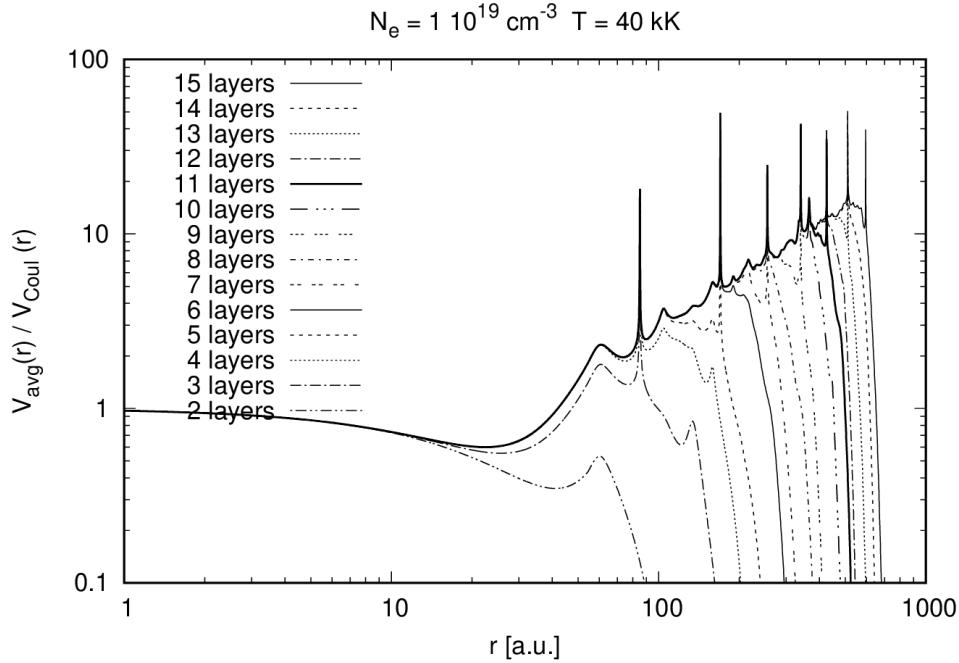


Figure 1. Convergence of the averaged potential with increasing number of ionic shells ($N = 1-15$). The calculations are done for $N_e = 1 \cdot 10^{19} \text{ cm}^{-3}$, $T=40\text{kK}$.

Figure 1 shows the systematic convergence of the potential towards the long range averaged plasma potential as more ion shells are included. The number of layers reflects onto the number of ions in a surrounding of considered, a middle one, that are influencing its potential. With the increasing of layers a ions count in a calculation of a averaged potential arises rapidly, so it was needed to estimate a optimal one. The characteristic slope change at $r \approx 50 \text{ a.u.}$ marks the transition from local ionic structure to collective plasma screening. It could be seen that no more than couple of layers should not be considered. As a figure of merit for hydrogen case we have used 8 layers. Further enlargement of considered ionic layers did not reflect on the quality of the results but only on wasted computing power. The rapid fall of the potential ratio at highest radii values is related to the size of the considered system and screening effect. It has only physical meaning that reflects only our finite size of considered ions like perturbers.

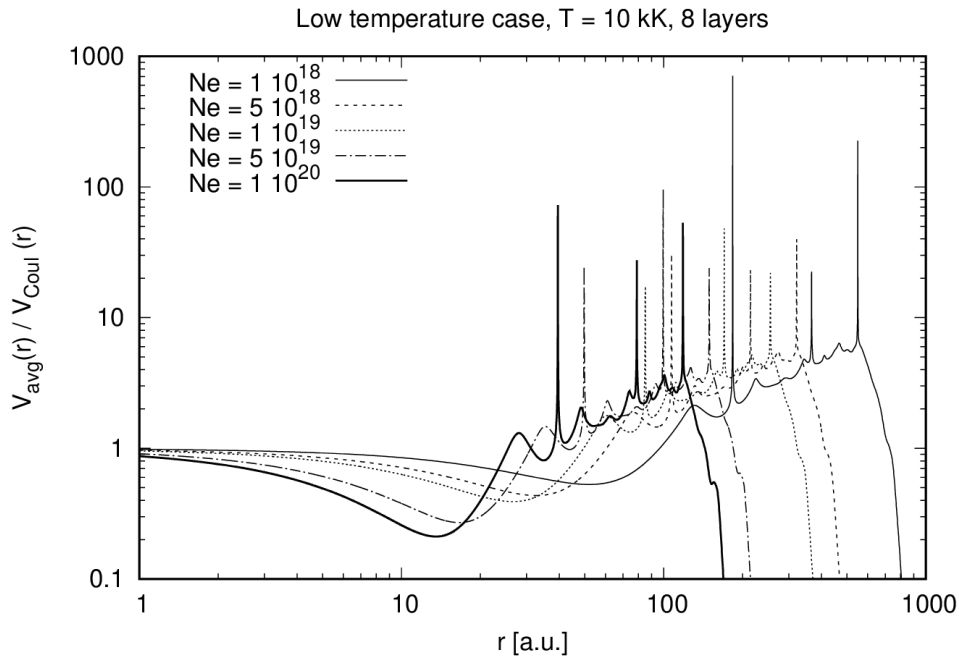


Figure 2. Low temperature case, $T=10kK$.

At lower temperatures (Fig. 2), shows the behavior of the calculated pseudo potential for low temperature $T=10kK$. The peak positions agree well with the expected FCC lattice positions (inset shows the radial distribution function).

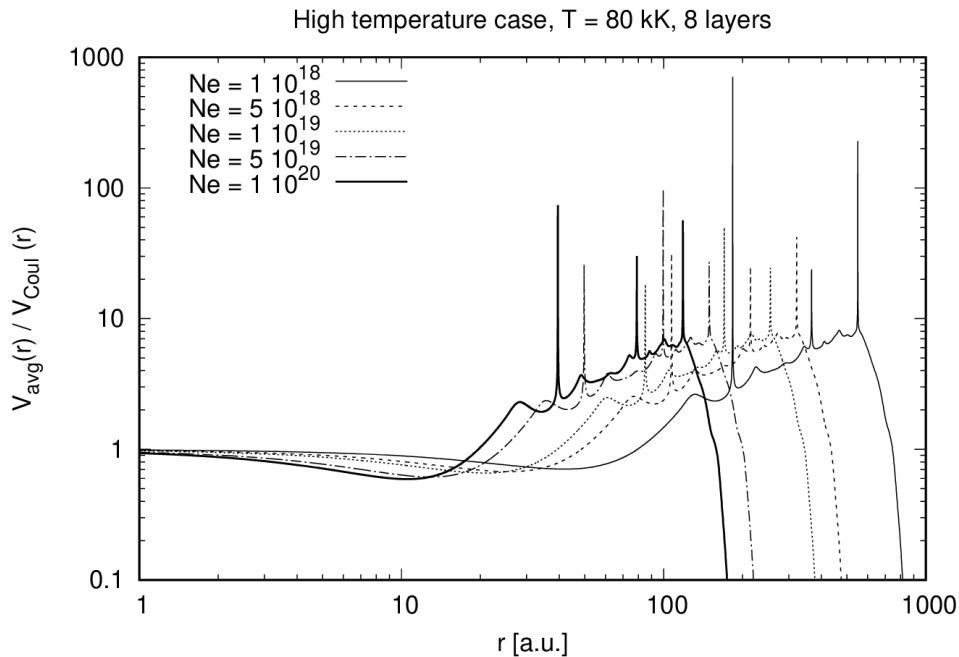


Figure 3. High-temperature case $T=80kK$, considered shells influence is a significantly broadened if compare with Fig. 2.

The high-temperature results (Fig. 3) differs from low temperature case, Fig. 2, solely by the electron screening. From here it could be denoted that the modified Yukawa potential screening has influence that may reflect onto the solution of the Schrodinger equation for the calculated potential.

3. Conclusions

Our analysis of screening in moderately coupled plasmas ($0.3 \leq \Gamma \leq 3$) yields three principal findings.

As first the modified Yukawa screening represents a improvement over conventional approaches in matching ab initio potentials with pure Debye screening, particularly for $1 \leq \Gamma \leq 2$ where correlation effects are strongest.

These results validate the pseudo crystalline approximation for solar chromosphere conditions (typically $\Gamma \approx 1.5$), while suggesting necessary modifications for photospheric applications ($\Gamma \approx 0.8$). The method's accuracy could be further improved by incorporating temperature effect of the ions, anisotropic screening at high Γ , adding quantum corrections for $T < 5$ kK, extending to multi-component plasmas.

This method is promising in obtaining a calculation intensity relaxed model that is capable producing a pseudo potential for describing a plasma behavior by the means of optical as well as transport cross sections

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