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A Needle in a Haystack: The Leptocline.

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Abstract.

Studies of solar (and stellar) dynamos face a problem of utter complexity, i.e., the interaction of a turbulent plasma in the convection zone, associated with latitudinal rotation together with magnetic field acting in a highly stratified medium, covering wide ranges of spatial and temporal scales. Recent theoretical models for the solar dynamo in complex 3-D simulations highlighted changes of the Sun’s stratification, mainly in the upper zones, pointing the role of the leptocline, a shallow and sharp shear layer in the top ~8-10 Mm. Here, we give here a brief history of the circumstances that led to the discovery of this layer, characterized by a strong radial rotational gradient at mid latitudes and self-organized meridional flows. We give an overview of the physical solar parameters that originate in this layer: opacity, superadiabicity and turbulent pressure changes; the hydrogen and helium ionization processes; sharp decrease in the sound speed; probably an oscillation phase of the seismic radius associated with a non-monotonic expansion with depth; probably temporal changes in photospheric zonal and sectorial modes and their associated gravitational moments. Likely also the initial place of the solar wind escape. In addition, the leptocline may play a key role in the formation of the magnetic butterfly diagram. Such results should be the starting point of systematic further investigations of structure and dynamics in this layer, leading to a better understanding of the solar cycle.

Keywords: Solar structure; Solar rotation; Solar convection.

Introduction.

Recent studies focused on the physical conditions prevailing in the Sun's subsurface layers aimed to understand how the solar rotation varies in latitude, in depth and with time. Indeed, the radiative interior of the Sun and its convective zone, are separated, at a depth of around 0.7 $R_\odot$, by a thin layer ($\approx$ 0.05 $R_\odot$), called the tachocline (Spiegel & Zahn, 1992), which is known to be the seat of the differential rotation. From this depth, once the latitude variations are underway, one might expect a uniform variation up to the surface. But this is not the case, as helioseismology show. Significant changes occurred in a near-surface shear layer (NSSL), occupying around the 5% of the solar interior, and, within this zone, around 2% (i.e. approximatively 10 Mm), it is not impossible that a nonlinear alpha-omega dynamo could be operating. Thus, the velocity shear may convert a part of the poloidal magnetic field into the toroidal field (in addition to the global dynamo operating in the tachocline region), strongly implicating the magneto-rotational instability (Vasil et al., 2024). It is thus suggested that the solar dynamo starts from this very thin sub-surface shallow layer, that we named by analogy to the tachocline, “leptocline” (Godier & Rozelot, 2001), from Greek “leptos’, thin and “klino”, tilt, or slope.

The aim of this paper is to briefly highlight the role of the leptocline.

A serendipitous discovery, albeit unnoticed.

Interpreting the solar limb shape distortions, Armstrong & Kuhn (1999) computed the density d_n and pressure s_n surfaces using a standard solar rotation profile, *n* being the order in a Legendre polynomial expansion of the contour of a shell of radius *r* (note that r < R⊙ is the radius of a thin shell enclosing a mass *dM*-or density $d\rho$). Figure 1 shows the results in the case of *n*=4 (hexadecapolar term). The tachocline, at *r* = 0.7 R⊙ is well marked, and a zoom for abscissa *r* between 0.960 R⊙ and 1 R⊙ shows a non-constant zone marked by a rise followed by a dip. This feature, overlooked at the time is the signature of the NSSL (near surface shear layer), obtained with a non helioseismic method.

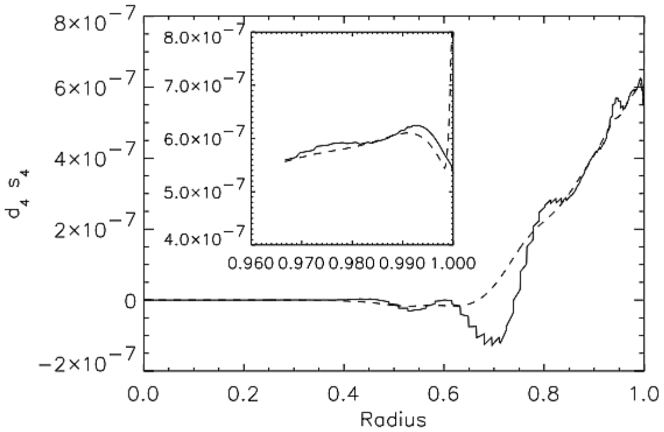


Figure 1. *The density (d) and pressure (s) surfaces variations with depth (radius), represented by the solid and dashed lines, respectively. After Armstrong & Kuhn (1999). The tachocline is well marked, and so is the leptocline (zoom in the upper figure), a feature that went unnoticed at the time.*

Solar rotation: a complex mechanism.

Solar rotation has been widely debated in recent years, through many books and articles, and can be summarized as follows (Figure 2):

- Below 0.68 R⊙, the radiative interior rotates almost rigidly, at about 430 nHz. The core itself may rotate faster.
- The transition from uniformly rotating radiation zone to differentially-rotating convection zone occurs in a thin layer from 0.68 R⊙ to 0.73 R⊙, called the **tachocline.**
- In the bulk of the convection zone (0.73 R⊙ < r < 0.96 R⊙), the rotation rate varies strongly with latitude. The equator rotates about 30% faster than the poles; from ~ 460 nHz at 0° latitude to ~ 340 nHz at 80° latitude.
- In a shallow layer > 0.96 R⊙ up to 1 R⊙, the rotation rate decreases by about 5% at all latitudes, showing however a more complex behavior near the surface. This layer is called the **Near Subsurface Shear Layer (NSSL).**
- A substructure of NSSL, so-called **leptocline**, located from the surface, covers about 10 Mm in depth within the convection zone (0.985 R⊙ < r < 1.0 R⊙).

- The contours of constant angular velocity are inclined by about 25° with respect to the rotational axis over a wide range of latitudes, i.e. rotation does not follow the Taylor-Proudman theorem[1].
- Within the leptocline unfolds an intricate behavior of the variation of the radial gradient $\partial \log \Omega / \partial \log r$, in latitude, depth and in time.

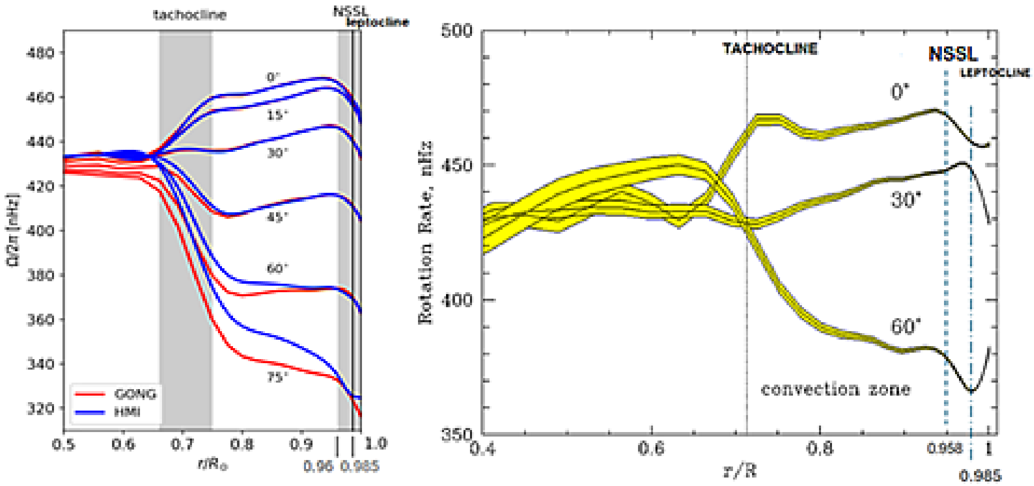


Figure 2. *Left. Internal profile of the solar rotation showing in grey shades (a) the tachocline (0.68 $R_\odot$ to 0.73 $R_\odot$); (b) the Near Surface Shear Layer (NSSL) beyond 0.96 $R_\odot$ (see Hotta et al., 2023) within which lies the leptocline (0.985 $R_\odot$ to 1.0 $R_\odot$). Results obtained by the Global Oscillation Network Group (GONG) (data courtesy of R. Howe, using the method of Howe et al. 2005) and the Helioseismic and Magnetic Imager (HMI) onboard the Solar Dynamics Observatory (SDO) (Larson and Schou, 2018). Note the sharp decrease of the velocity rate. Right. Solar profile after Kosovichev et al. (1997). The complex profile is clearly visible within the NSSL, sharply pronounced in the leptocline, where the rotation gradient displays large and significant variations in latitude, depth and time (cf. para 6).*

Solar radius with respect to the leptocline.

An accurate measure of the solar diameter[2] is still very difficult. The first reason is to agree on the definition of a diameter for a rotating fluid; the second is to highlight the physical reasons that have emerged between theoretical models and ground-based or space observations. A review of photospheric radius measurements R_{ph} has been made in Rozelot & Damiani (2012, upgraded in Rozelot et al. 2016), to which eclipse measurements can be supplemented (Lamy et al. 2015). Based on the work of Takada & Gough (2024), and to make things here easier, we

[1] Under certain conditions, when a solid body is moved slowly within a fluid that is steadily rotated with a high angular velocity Ω, the fluid velocity will be uniform along any line parallel to the axis of rotation. Ω must be large compared to the movement of the solid body in order to make the Coriolis force large compared to the acceleration terms.

[2] Radius and diameter are used interchangeably, with a few exceptions. For instance, in the case of observations by solar astrolabes, where the diameter is not twice the radius, a very small angle being introduced by construction between the measurement of the two opposite radii.

propose the following glossary as given in Table 1 (see also Kosovichev et al., 2025, for further details).

Table 1. *Proposed new glossary of the various solar radius in use (in brief). Modified from Takada & Gough (2024).*

Radius name	Label	Meaning
Photospheric	R_{ph}	Distance from the solar center to the photospheric surface, defined as the layer where the optical depth $\tau = 1$ for a particular wavelength, usually 500 nm.
Canonical	R_c	Adopted radius through a consensus, generally, at IAU GA. The radius used to calibrate models is labelled $R_\odot$
Seismic:	R_s	Radius calculated using helioseismology. In general, the radius is defined as the radial distance at which the temperature equals the effective temperature.
1. Fundamental	R_f	Fundamental photospheric radius scaled through f-modes, which is the distance from the solar center to the center of energy of each f-mode (essentially the peak in the kinetic-energy distribution).
2. Acoustic	R_{ac}	Acoustic photospheric radius scaled through p-modes, which is the distance from the center to the subphotospheric layer where the acoustic cut-off frequency changes extremely rapidly.

The two space missions SOHO (Solar and Heliospheric Observatory) (Scherrer et al., 1995) and SDO (Solar Dynamics Observatory) (Scherrer et al., 2012) has provided long time-series of solar oscillations that has been intensively analyzed. It has been shown that the radius of the Sun can be determined, that can be called the "seismic radius" of the Sun, to differentiated it from the conventional photospheric radius. Schou (1997) & Antia (1998) were the first in analyzing the data to find a "fundamental photospheric" radius calibrated through f-modes R_f, lower than the observed one through optical instruments (photospheric radius R_{ph}): by 310 km for the first author and 203 km for the second. This discrepancy seems to have been explained by Haberreiter et al. (2008) by a difference of 0.333 ± 0.008 Mm between the height at disk center where the optical depth at $\tau_{500} = 1$, and the inflection point of the intensity profile on the limb. However, Takada & Gough (2024) point out that the three models used by Haberreiter et al., (2008) close to those of Brown & Christensen-Dalsgaard (1998), "*do not lead to the same results, rendering this explanation premature*". Further effort to study this issue is required to clarifying the properties of the NSSL.

Kosovichev & Rozelot (2018), analyzed 21 years of helioseismology data (the Michelson Doppler Imager (MDI) on board SOHO - Scherrer et al. 1995) and the Helioseismic and Magnetic Imager (HMI on board SDO - Scherrer et al. 2012) to resolve previous uncertainties and compare variations of the seismic radius over two solar cycles. After removing the f-mode frequency changes associated with the surface activity, they find that the mean seismic radius is reduced by 1–2 km during the solar maxima and that most significant variations of the solar radius occur beneath the visible surface of the Sun at a depth of about 5 ± 2 Mm, where the radius is reduced by 5–8 km (Figure 3). Other authors have reached similar conclusions, see for instance Hernandez et al. 2009; Antia (2003) who reported "From our results, we can put a conservative upper limit of 2 km on radius variations during the last 6 yr, from 1996 May 1 to 2002 August 21", (but with a gap in data sets between 1998.5 and 1999.2).

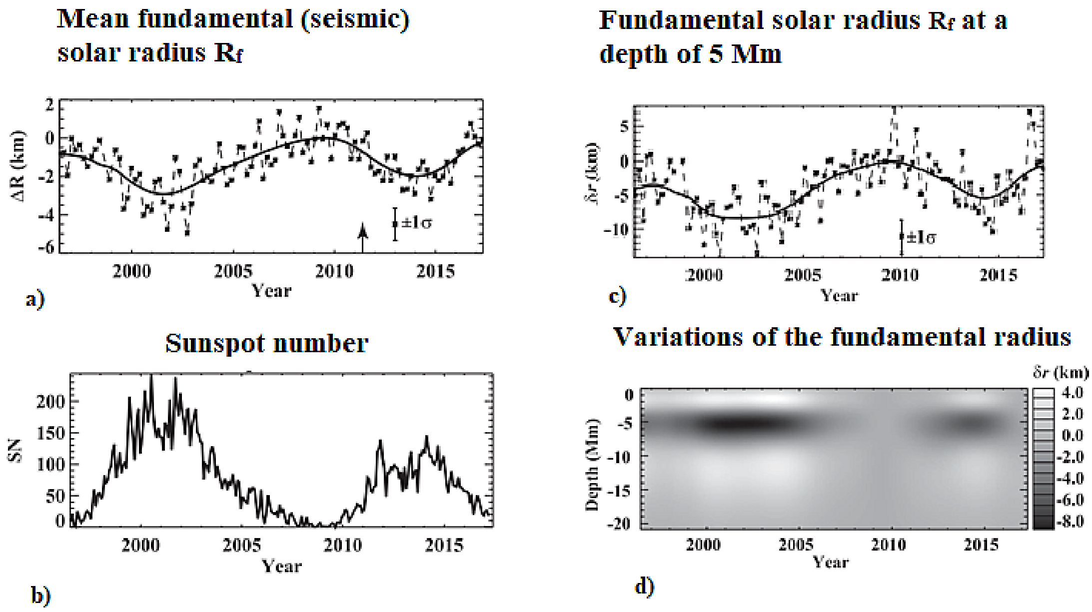


Figure 3. *Solar cycle variations of (a) the mean fundamental seismic radius R_f (average over all values of degree ℓ, from ℓ = 137 to ℓ = 300); the arrow indicates the start of the HMI data set (after MDI data set); (b) the sunspot number averaged for the same time intervals as the helioseismology data: antiphase correlation is clearly visible; (c) changes of the fundamental seismic radius R_f at the depth of 5 Mm; (d) variations with time and depth beneath the solar surface. The dark color (negative values) corresponds to contraction and the light color (positive values) corresponds to expansion of subsurface layers (After Kosovichev & Rozelot, 2018).*

Such variations may seem insignificant and questionable in view of several factors: data errors (a change in systematic errors may lead to spurious variations of the same order), magnetic field variations below the outermost surface layers, density perturbations, turbulence, supergranulation… However, HMI has proved to be an extremely high precision instrument, able to measure the oscillations of the solar limb with a positional accuracy of the order of microseconds and a relative brightness accuracy of 10^{-6}. Figure 3 (a, b), show an antiphase correlation of the fundamental surface photospheric radius scaled through f-modes. It can be noticed that radius variations give a real insight into changes of the Sun's subsurface stratification, particularly around an average depth of 5 Mm (between 2 and 8 Mm, i.e. inside the leptocline). Kholikov & Hill (2008) using very low-p-modes found also a clear antiphase variation of the acoustic photospheric radius R_{ac} with solar activity.

Such results have been obtained assuming a homogeneous stratification. What happens if it is not the case? Assuming variations of δr/r, Lefebvre and Kosovichev (2005) and Lefebvre et al. (2007a, 2007b) found a change in the structure of the subsurface layers down to about 97 % of the photospheric radius. Even if (i) the contribution of the near-surface effects have been neglected, and (ii) the near-surface effects of turbulence and magnetic fields were not treated, Figure 4 (left) shows (i) no significant changes in the variation of the subsurface layers' depth below 0.97 $R_\odot$, and (ii) nonmonotonic changes in the stratification, with the inner layer (below 0.99 $R_\odot$); moving up during the increase of activity (compression) while the outer layer (above 0.99 $R_\odot$,) moves down (relaxation), giving substance to the leptocline. The precise localization could be ascertained by about less than 0.003 $R_\odot$.

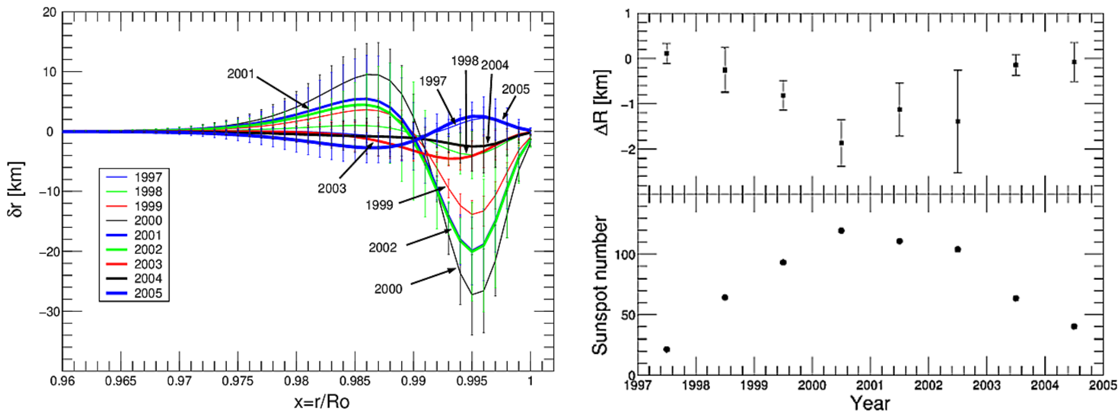


Figure 4. *Left: Radial variation δr (in km) of the solar stratification, as a function of the fractional radius x = r/ $R_\odot$, obtained as a solution of the inversion of f-mode frequencies by a least-squares regularization technique (Lefebvre & Kosovichev, 2005). The reference year is 1996. The error bars are the standard deviation after averaging over a set of random noise added to the relative frequencies. The leptocline is well localized between 0.985 and 0.996, with a typical half-width of ∼ 0.003 $R_\odot$. Right: temporal variation of ΔR near the solar surface at r = $R_\odot$, plotted together with the variation of the sunspot number for the same period. The variation of the seismic radius at the surface is found to be in antiphase with the solar cycle, with an amplitude of about 2 km. It is important to keep in mind that without information at high degree ℓ we cannot constrain the surface radius better and that, in reality, this variation at the surface could be larger provided if it is more localized.*

Figure 4 (right) shows temporal variation of the radius near the solar surface (R_{ph}), together with the variation of the sunspot number for the same period. The variation of the seismic radius at the surface is in antiphase with the solar cycle, with an amplitude of about 2 km. All these results are nevertheless subject to the fact that the lack of very high f-mode degrees prevents to understand the very outer layers (above ∼3 Mm). One might think that it would be enough to expand the degree ℓ to infinity to converge R_f to R_{ph}, but then, the surface is undefined (a fantom, as Takada & Gough, 2024 showed).

5. Rotational gradient with respect to the leptocline.

Komm (2022) made an exhaustive study of the radial gradient of the solar rotation rate in the near-surface shear layer (NSSL), from about 0.950 $R_\odot$ to the solar surface, using the ring-diagram analysis applied to Global Oscillation Network Group (GONG) and Helioseismic and Magnetic Imager (HMI) Dopplergrams. He thus explored the variation of the radial gradient, over latitudes, density, depth and time (Solar Cycles 23 and 24). The results, which are very extensive, can be summed up as follows (see also Figures 5 and 6):

- The average radial gradient is $\partial \log \Omega/\partial \log r = -1.0 \pm 0.1$ at a depth of 0.990 $R_\odot$ (in agreement with previous studies);
- The average radial gradient is rather flat within ± 40° latitude from about 0.970 $R_\odot$ to the solar surface;
- The average radial gradient is $\partial \log \Omega/\partial \log r = -0.11 \pm 0.01$ at 0.950 $R_\odot$, (base of the NSSL), increasing in amplitude to -0.42 ± .02 at 0.97$R_\odot$ and then to −1.04 ± 0.06 at 0.990$R_\odot$;
- Between 0.990$R_\odot$ and 0.998$R_\odot$, the average radial gradient is steeper than -1 within the observed latitude range:
  - (i) the steeper slope $\partial \log \Omega/\partial \log r = -2.6 \pm 0.2$ is obtained at 0.998 $R_\odot$ (or 1.5 Mm in depth), derived from HMI data; a radial gradient steeper than −2 close to the surface agrees with recent global results using a new technique that allows to fit high degree modes (as previously seen, Reiter et al., 2020);
  - (ii) such high radial gradient indicates that the NSSL consists of two separate regimes. The first one, near the surface, coincides with a shallow shear layer within 10 Mm from the surface (leptocline), as found in a numerical model of the NSSL with radial gradients far steeper than −1 (Kitiashvili et al., 2022);

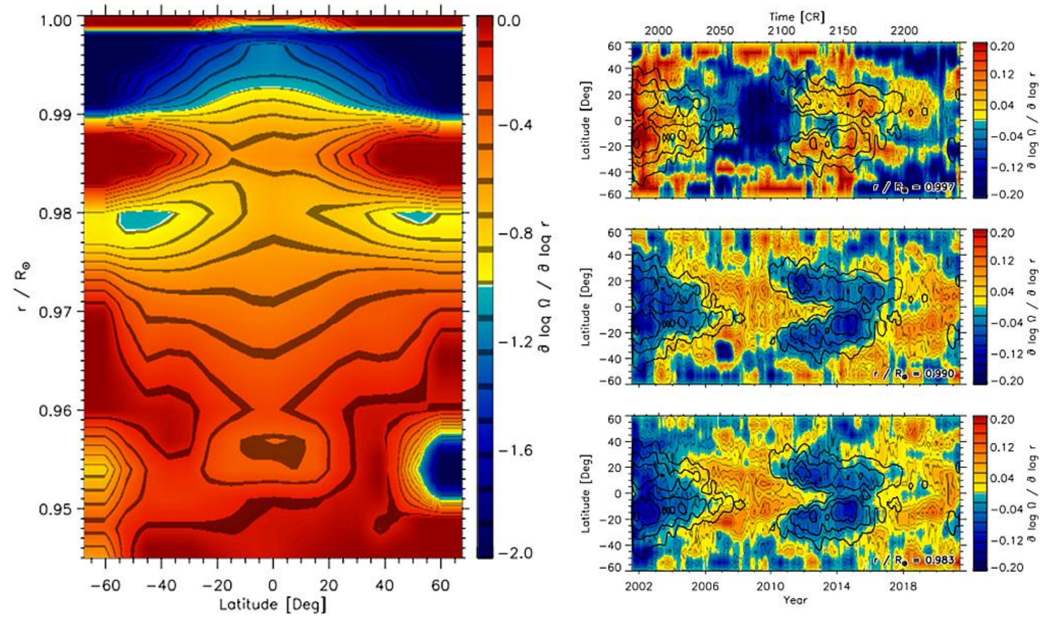


Figure 5. *Left: The radial gradient of the solar rotation rate as a function of latitude and radius using 30°-tile Ring Diagram Analysis (RDA) results derived from HMI data averaged over Solar Cycle 24. The values have been interpolated on an equidistant grid in radius and onto the 7.5° grid in latitude used by 15°-tile RDA.*

Right: The temporal variation of the residual gradient of the solar rotation rate after subtracting the average offset and subtracting the temporal mean at every latitude at three depths (top: 0.997$R_\odot$, middle: 0.990$R_\odot$, bottom: 0.983$R_\odot$). The time coordinate is given in years (bottom x-axis) and

Carrington Rotations (CRs, top x-axis). Black contours indicate magnetic activity (5, 10, 20, 40 G) smoothed over five CRs. A variation with the solar cycle is noticeable at all three depths. The residual gradient is larger (in amplitude) than average at locations of high magnetic activity at 0.990R⊙ and 0.983R⊙ and smaller than average at quiet locations. However, it is the opposite behavior at 0.997R⊙ with the residual gradient being smaller (in amplitude), than average at locations of high magnetic flux. (See Komm, 2022).

(iii) a value of −1.5 occurs on average at 0.996R⊙ (or about 3 Mm in depth), which coincides with the maximum of the flow divergence of quiet regions, which represents supergranular flows; the divergence decreases to half its maximum amplitude by a depth of about 8 Mm, which might imply that the upper layer of the NSSL is heavily influenced by supergranule;

- The radial gradient of the solar rotation rate varies with the solar cycle:
  At locations of high magnetic activity, the radial gradient is more negative than average, from about 0.970 R⊙ to 0.990 R⊙, while in quiet regions the radial gradient is less negative than average at these depths. Close to the surface, at 0.997 R⊙, this relationship appears to be reversed.
- The variation of the radial gradient more likely indicates the presence or absence of magnetic flux (above a certain threshold).

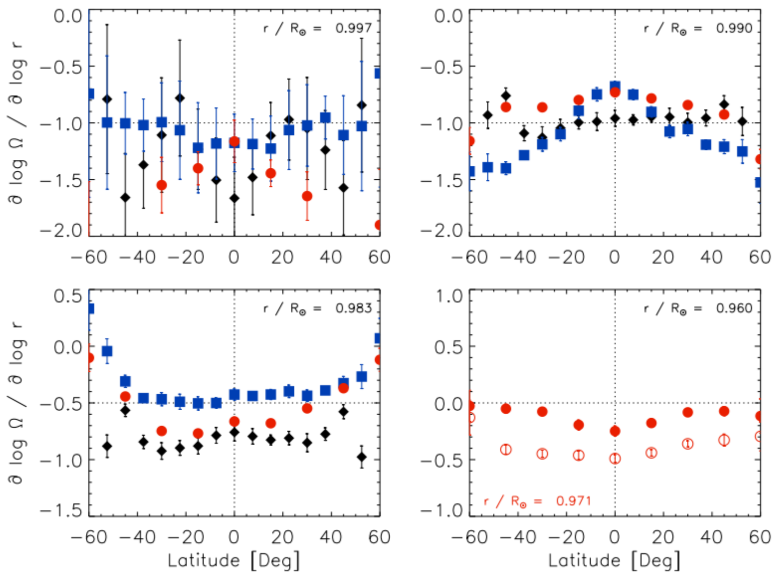


Figure 6*. The radial gradient of the solar rotation rate as a function of latitude at four depths (top-left: 0.997R⊙, top-right:0.990R⊙, bottom-left: 0.983R⊙, bottom-right: 0.960R⊙), using 15°-tile Ring Diagram Analysis (RDA) (black diamonds: GONG, blue squares: HMI) and 30°-tile RDA (red circles: HMI) averaged over Solar Cycle 24. The open symbols indicate values at 0.971R⊙ (bottom-right). (See Komm, 2022).*

Attempts to modelize the NSSL.

From a theoretical point of view the influence of magnetic fields on incident acoustic waves is a complex phenomenon and there is still a need to better understand all the physical processes involved. However, progress in numerical simulations provide new insight in that field. Kitiasvilli et al. (2023) analyze realistic 3D radiative hydrodynamics simulations of solar subsurface dynamics in the presence of rotation in a local domain 80 Mm wide and 25 Mm deep, located at 30° latitude. The simulation results reveal the development of a shallow 10-Mm deep substructure of the Near-Surface Shear Layer (NSSL), characterized by a strong radial rotational gradient and self-organized meridional flows. This shallow layer ("leptocline") is located in the hydrogen ionization zone associated with enhanced anisotropic overshooting-type flows into a less unstable layer between the H and He II ionization zones. Their results, shown in Figure 8, reveal a significant decrease in the azimuthal rotation velocity with depth by 38 m/s in a 2 Mm deep layer below the photosphere. Below 7 Mm, the rotation rate is slower than the imposed mean rotation rate by about 5 m/s. The rotation rate with depth is not uniform: it increases by $6 - 7$ m/s per Mm from the sub-photospheric layers to about 4 Mm below, while below 4 Mm, the flow accelerates by about 2 m/s per Mm. So, they identified a 10-Mm thick near-surface shear layer (the leptocline), clearly visible in the relative differential rotation profile. Moreover, their study shows strong negative values of the gradient of rotation, about -4 in subsurface layers, and an increase in the deeper layers (Figure 7). They also evidenced that the outer layers of the NSSL form a distinct substructure characterized by enhanced turbulent convection and strong rotational shear. So, we can argue that the leptocline constitutes the upper part of the Near-Surface Shear Layer, of about 10 Mm thick. The interface between the leptocline and deeper layers is characterized by overshooting downdrafts, which may intensify the turbulent mixing below this layer.

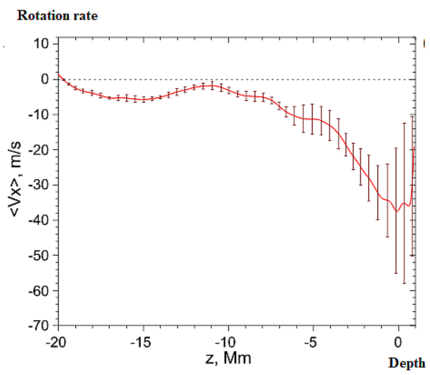


Figure 7. *Theoretical mean radial profiles of deviations of the azimuthal flow speed from the imposed rotation rate at 30° latitude (5m/s). Note the change in the 10 Mm depth below the surface, within the leptocline. Kitiashvili et al. (2023).*

Conclusion.

As pointed out by Reiter et al. (2015) local and global helioseismology[3] has proven to be extremely powerful tools for the investigation of the internal structure and dynamical motions of the Sun. Their combined outcomes show that the structure of the Near Surface Shear Layer

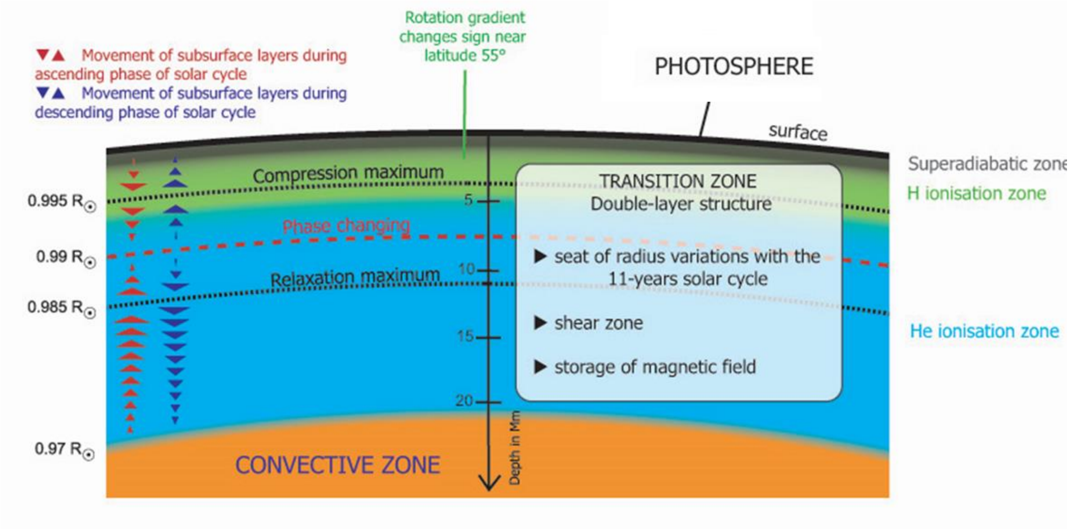
[3] The term "global helioseismology" refers either to studies that employ the low- and intermediate-degree p-modes whose lifetimes are truly long enough for them to be globally coherent, or to studies that employ spherical harmonic decompositions that are computed from nearly the entire visible solar hemisphere. Studies that do not use modes that have such long lifetimes or are computed from observations that cover much smaller portions of the visible hemisphere are considered to employ the tools of "local helioseismology" (Reiter et al. 2015).

is now well-resolved, especially for the subsurface rotational shear layer, the leptocline. Results also suggest that the heat transport properties may be significantly different from the predictions of the mixing-length theory.

It can be advanced that the frequency shifts observed in high-degree p-modes (characterizing conditions in the shear layer just below the solar surface) are caused by a combination of strong fields present at the tachocline and weak turbulent fields present in the near-surface shear layer (Kiefer & Roth, 2018). Such an issue has been confirmed by Baird et al. (2024) but should be explored in greater detail in the future.

Hemispheric asymmetry observed in sunspots (and other solar activity proxies) is an important aspect of solar cycles as it provides important clues for understanding dynamical processes in the interior of the Sun. Inceoglu et al. (2017) found that the two hemispheres are decoupled when comparing separately their total magnetic energy. Baird et al. (2024) confirmed that the magnetic activity in each hemisphere evolves independently. As solar activity decreases with time, the amplitude of frequency shifts is also found to decrease in both hemispheres, but more rapidly in the southern hemisphere, indicating that frequency shifts follow this evolution closely. The role of the leptocline in such findings is also certainly to be clarified.

Lastly, the study of the gradient of rotation varies from close to zero at 0.985 $R_\odot$ to steeper than −1 at 0.978 $R_\odot$. The decrease toward zero with greater depth is the same within about 20° latitude, while at latitudes poleward of 40° this decrease is not monotonic. It appears to reverse close to the surface at 0.997 $R_\odot$, indicating that the dominant processes differ at the base of the NSSL and in the leptocline. It also varies with time and could be used as a precursor to detect changes of the solar cycles. Moreover, the change in the steepness of the radial gradient below and above 0.989$R_\odot$ (rapid increase toward the solar surface from 0.986 $R_\odot$ to 0.998 $R_\odot$, and slower decrease with increasing depth below this range) shows a "saddle" point near 0.989 $R_\odot$. This point has been already detected in the non-monotonic variation with depth of the solar radius (see Figure 6). This issue may confirm that the leptocline is constituted of two regimes, that is beginning to be clearly modelized in 3-D.



Schematic view of the subsurface layers, transition zone between the convective zone and the photosphere (non-scale scheme). This zone is called "Leptocline" (from the greek "leptos" = thin and "klinô" = slope).

The NSSL is not a homogeneous shear layer. It can be divided at least in two regions; the first part and thickest region, extends from its base (0.958 $R_\odot$, about 30 Mm deep) to around 0.986 $R_\odot$ (10 Mm), and the second part, up to the surface, the leptocline, where the gradient of rotation exhibits highly complex variations in both amplitude and latitude. Based solely on these considerations, Soares et al. (2024) suggests that the leptocline could be could be divided again into two sub-layers: the M layer (for middle one), centered at 0.9965 $R_\odot$ (~2.4 Mm) and the S layer, beginning at 0.9977 $R_\odot$ (~1.6 Mm) up to the surface, being understood that the inversion results are unreliable at layers shallower than ∼1 Mm. However, the leptocline cannot be reduced to highlight the only properties of the rotational gradient as other many physical phenomena take place there (H and He ionization zones for instance), and the superadiabatic region being far to be understood.

As a general conclusion, it is not impossible that the solar-dynamo could take place in the leptocline. Results presented here could be the starting point of systematic further investigations of structure and dynamics in this layer, will lead to a better understanding of the solar activity.

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