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MHD Instabilities in Shear Flow of Anisotropic Solar Wind Plasma

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Abstract

The study investigates the stability of anisotropic collisionless plasma layers subjected to shear flows within the magnetohydrodynamic (MHD) framework. Using moment equations derived from the Vlasov kinetic model and incorporating heat fluxes along the magnetic field, a generalized wave equation is obtained to describe small perturbations in shearing plasma. The boundary value problem for a smooth hyperbolic velocity profile is solved under the WKB approximation to determine the complex spectral parameter governing instability growth. The resulting integral dispersion relation encompasses both body and surface modes of plasma instabilities. Analysis reveals that spatial confinement significantly modifies classical instability behavior: reducing the layer width amplifies mirror instabilities while suppressing oblique fire-hose modes. However, increasing the flow velocity gradient enhances oblique fire-hose instability, with body modes transforming into surface Kelvin–Helmholtz-type modes at high gradients. These results highlight the critical role of finite shear-layer geometry and velocity gradients in regulating instability development and turbulence generation in anisotropic cosmic plasmas, such as the solar wind and astrophysical jets.

Keywords: Solar Wind; MHD shearing flows; WKB solution

1. Introduction

In collisionless cosmic plasmas, the dissipation of macroscopic flow energy into microscopic scales remains one of the key problems of plasma astrophysics [Cranmer et al., 2015]. Energy transfer occurs through turbulent cascades that channel large-scale kinetic energy toward smaller, kinetic scales, where it is converted into heat by wave–particle interactions and nonlinear effects [Marsch, 2006; Gary et al., 2016]. Such processes underlie the heating and acceleration of the solar corona and solar wind [Kiyani et al., 2015; Verdini et al., 2019].

The onset of turbulence is closely related to plasma instabilities, which arise when free energy is stored in anisotropic pressures or velocity gradients. Shear flows are among the most efficient sources of such energy and are widely observed in the solar wind, magnetospheric boundaries, and coronal mass ejection fronts [Kivelson and Chen, 1995; Hasegawa et al., 2004; Foullon et al., 2011]. In these regions, the Kelvin–Helmholtz (KH) instability may develop, leading to large-scale vortices and enhanced transport across magnetic boundaries [Sisti et al., 2019].

However, weakly collisional magnetized plasmas are typically anisotropic with respect to the magnetic field, giving rise to instabilities such as fire-hose and mirror modes, which regulate pressure balance and redistribute energy [Vedenov and Sagdeev, 1958; Hellinger et al., 2006]. The classical CGL model [Chew et al., 1956] captures basic anisotropy effects but neglects parallel heat fluxes important in collisionless conditions. A more complete description is provided by the 16-moment fluid formalism, which includes anisotropic pressures and parallel heat flux dynamics [Ramos, 2003; Dzhalilov and Kuznetsov, 2013].

Previous studies have shown that finite shear-layer width can substantially modify or enhance these instabilities [Dzhalilov et al., 2011]. The present work extends this analysis by investigating the evolution of oblique fire-hose modes in anisotropic plasma flows with velocity shear. Using the 16-moment MHD framework, we derive the corresponding wave equations and apply the WKB approach to determine the spectral parameters and growth rates for different shear and layer thicknesses. The obtained results help to clarify the transition from ordered shear flows to turbulence in space plasmas such as the solar wind and coronal structures.

2. Basic Equations in the Fluid Description

For the fluid description of a collisionless anisotropic plasma aligned with an external magnetic field, the extended fluid model provides a more complete system than the classical CGL (Chew–Goldberger–Low) approximation, as it includes the evolution of heat fluxes along the magnetic field [Oraevski et al., 1985; Ramos, 2003]. This formulation captures the effects of anisotropy and dispersion essential for describing collisionless plasma dynamics [Ramos, 2003; Dzhalilov and Samadov, 2024]. In the present analysis, a simplified version of the extended MHD system is employed, incorporating both ion and electron contributions. Because electron–ion coupling is weak in collisionless regimes ($m_e/m_i \ll 1$), the large-scale dynamics are governed mainly by ions, while electrons maintain quasi-neutrality. The plasma flow is assumed plane-parallel and directed along the z-axis, with a velocity shear in the x-direction. The equilibrium velocity is given by $v_0 = (0,0,V_0(x))$, and the background magnetic field B_0 is uniform and parallel to z. The equilibrium parameters $\rho_0, p_{\perp 0}, p_{\parallel 0}, B_0, S_{\perp 0}, S_{\parallel 0}$ are taken to be constant.

The stability of this configuration is analyzed by introducing small perturbations to all physical quantities, $f = f_0 + f'(x,y,z,t)$, where $f'(x,y,z,t) \sim F(x)e^{i(k_y y + k_z z - \omega t)}$. This Fourier decomposition with respect to $y, z, t$is crucial because the coefficients of the resulting differential equations depend only on x. Consequently, this step transforms the coupled system of partial differential equations into a set of ordinary differential equations (ODEs) in x.

Here, ω is the wave frequency, and k_y, k_z are the wavenumber components in the plane perpendicular to the shear direction. After linearizing the 16-moment equations and substituting the perturbed quantities, the system reduces to a single second-order differential equation for the magnetic field perturbation $B_x(x)$:

$$\frac{\partial}{\partial x}\left(A(x)\frac{\partial B_x}{\partial x}\right) - \beta_A(x) B_x = 0. \qquad (1)$$

This is the fundamental wave equation governing the spatial evolution of the magnetic perturbation. Where;

$$A(x) = \frac{\beta_A \beta_*}{k_y^2 \beta_* + k_z^2 \beta_A}, \qquad (2)$$

$$\beta_A(\tau, \Omega_A) = \alpha + \beta - 1 - \xi(\tau, \Omega_A)^2, \qquad (3)$$

$$\beta_*(\tau, \Omega_A) = \beta + 2\alpha + \frac{2\alpha^2\,(\xi^4 + 2g\,\xi^3 + 2g^2\,\xi^2 - 5\xi^2 - 6g\,\xi + 3)}{(\xi^2 - 1)\,(\xi^4 - 6\xi^2 - 4g\,\xi + 3)}\,. \qquad (4)$$

$$\xi(x) = M[\Omega + \Delta \tanh(\sigma x)], M = \frac{\bar{V}_0}{c_\parallel}, \Delta = \frac{h-1}{h+1} \qquad (5)$$

$\Omega = \Omega_r + i\Omega_i$ is the complex spectral parameter to be determined from the boundary conditions, M is the Mach number and Δ characterizes the shear strength ($0 \leq \Delta \leq 1$).

The coefficients in the obtained second-order ordinary differential wave equation (1) are variable and appear as complex functions of the background velocity profile $V_0(x)$. Since $V_0(x)$is, in general, an arbitrary function of the spatial coordinate x, its specific form determines both the structure and stability properties of the plasma flow. When two distinct plasma streams are in contact, a finite transition region inevitably develops between them. The thickness of this transition layer is governed by the relative velocities and the physical parameters of the adjoining flows.

To model this configuration analytically, we adopt a smooth velocity profile expressed through a hyperbolic tangent–type function:

$$V_0(x) = \frac{V_{02}e^{\sigma x} + V_{01}e^{-\sigma x}}{e^{\sigma x} + e^{-\sigma x}}, \sigma \geq 0. \qquad (6)$$

Here, $V_0(-\infty) = V_{01}$and $V_0(+\infty) = V_{02}$represent the asymptotic flow velocities, while $h = V_{01}/V_{02} \geq 1$and $V_0(0) = (V_{01} + V_{02})/2 = \bar{V}_0$denote, respectively, the velocity ratio and the mean flow speed. The parameter σ characterizes the inverse thickness of the shear (transition) layer, denoted by L.

Figure 1 schematically illustrates several possible velocity profiles $V_0(x)$for different values of σ. As the dimensionless product σLincreases, the transition region between the two streams becomes narrower, and for $\sigma L \gg 1$, it effectively approaches a sharp velocity discontinuity separating $V_{01}$and V_{02}. Conversely, when $\sigma L \ll 1$, the transition region broadens considerably, corresponding to a smooth, gradual shear between the flows.

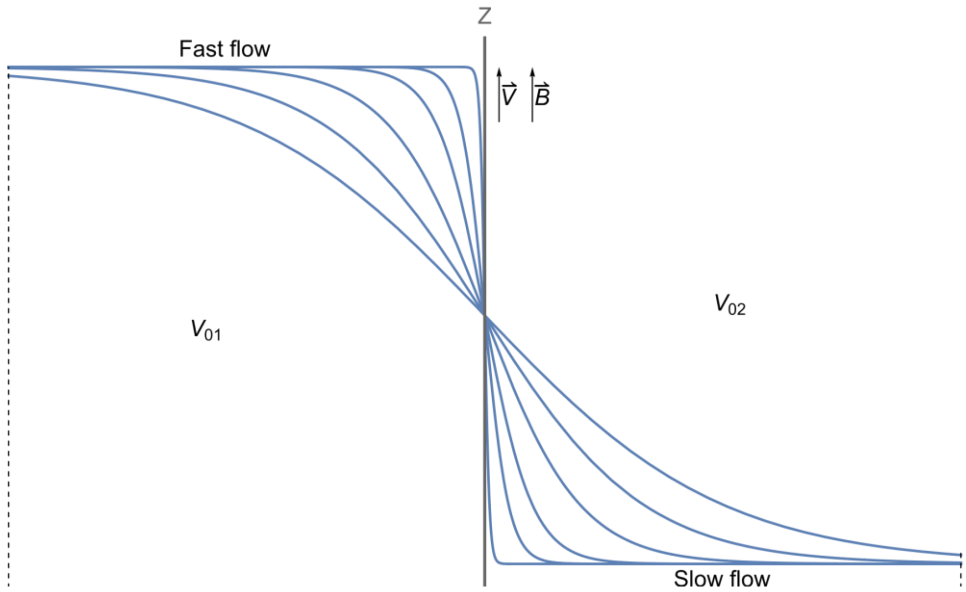

Figure 1. *Schematic representation of MHD plasma shearing flow. Different line profiles correspond to different values of* σL *in (6).*

3. WKB Approximation and Mathematica Solution

To solve the boundary value problem described above, the Wentzel–Kramers–Brillouin (WKB) approximation was applied, which allows analytical simplification of the second-order wave equation with slowly varying coefficients. As shown in [Dzhalilov and Samadov, 2024], the dispersion relation obtained from this approach can be expressed in the form

$$\int_0^1 (\sqrt{f(\xi_+)} + \sqrt{f(\xi_-)})\, d\tau = \lambda_n,$$

or equivalently,

$$\int_0^b \frac{\sqrt{f(\xi_+)} + \sqrt{f(\xi_-)}}{1 - z^2}\, dz = \sigma_L \lambda_n, \qquad (24)$$

where

$$f(\xi) = -\frac{(1-l)\beta_*(\xi) + l\beta_A(\xi)}{\beta_*(\xi)}, \lambda_n = \frac{n\pi}{kL} = \frac{\lambda_\perp n}{2L}. \qquad (25)$$

Here, $\lambda_n$represents the ratio between the wavelength $\lambda_\perp$in the (y, z)plane ($k^2 = k_y^2 + k_z^2$, $k = 2\pi/\lambda_\perp$) and the geometric width $2L$ of the plasma layer. The integer n denotes the number of nodes of the eigenfunction along the x-axis. If we define the longitudinal scale of the fluctuation structures as $\lambda_x = 2L/n$, then $\lambda_n = \lambda_\perp/\lambda_x$.

To determine the complex eigenfrequencies $\Omega = \Omega_r + i\Omega_i$ satisfying the dispersion equation (24), numerical computations were carried out in *Mathematica*. Specifically, the functions were integrated using NIntegrate, and the corresponding roots of the dispersion equation were obtained with the FindRoot command. This computational scheme provides discrete complex values of Ω whose imaginary parts determine the growth rates of unstable fire-hose modes in shear flows of anisotropic plasmas.

4. Results and Conclusions

The dispersion relation derived from the WKB approximation was solved numerically to obtain the complex frequency $\Omega = \Omega_r + i\Omega_i$, where the imaginary part $\Omega_i$characterizes the instability growth rate. The results are shown in Figs. 2–3 for different values of the shear parameter Δ, the layer thickness σ_L, and the structural parameter λ_n[Dzhalilov and Samadov, 2024].

Figure 2 shows the dependence of $\Omega_i$on the transverse scale parameter $\lambda_n$at fixed $M\Delta = 1.5$. The instability is strongest for small λ_n(few-node body modes) and decreases steeply as $\lambda_n$grows, demonstrating that modes with fine transverse structure couple more efficiently to the shear. Increasing $\sigma_L$shifts the instability onset to higher $\lambda_n$and increases its magnitude, confirming that narrower layers favor stronger growth.

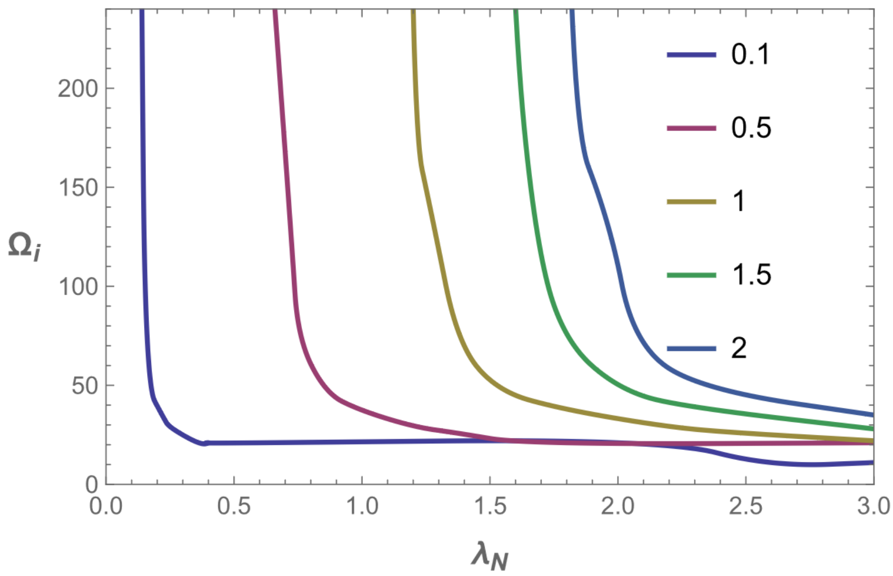

Figure 2. *The dependence of instability growing rate M Ωi of fire hose modes on the λn at different values of σL (numbers at the curves) when M Δ = 1.5.*

Figure 3 illustrates how the growth rate $\Omega_i$varies with the shear amplitude Δat fixed $\lambda_n = 1$. For weak shear ($\Delta \lesssim 0.1$), the instability is nearly suppressed. As Δincreases, $\Omega_i$rises sharply and reaches a maximum around $\Delta \simeq 0.2 - 0.3$, indicating efficient energy transfer from the mean flow to the perturbations. Beyond this range, the growth rate gradually decreases. The effect strengthens for larger σ_L, showing that a sharper velocity gradient enhances the instability by concentrating free kinetic energy in the transition layer.

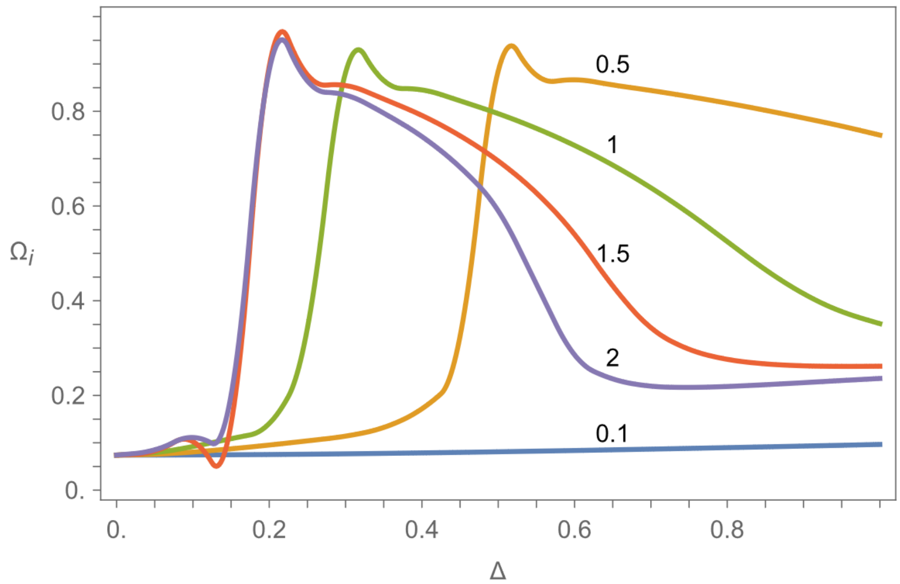

Figure 3. *The dependence of instability growing rate Ωi of fire-hose modes on the shearing rate of plasma supersonic flows Δ at different values of σL (numbers at curves) when λn = 1.* *Overall, the analysis shows that the fire-hose instability in anisotropic plasmas becomes highly sensitive to both the velocity-gradient scale and flow anisotropy. Sharper shear layers ($\sigma_L \gtrsim 1$) and moderate velocity contrasts ($\Delta \sim 0.2 - 0.4$) yield the largest Ω_i, while overly wide or weakly sheared flows remain stable. These results suggest that in space plasmas—such as solar-wind shear regions or CME boundaries—fine velocity gradients can efficiently trigger wave amplification and local turbulence, bridging the transition from bulk flow to chaotic magnetic structures.*

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