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Applications of Chaos Theory to Astrophysical Time Series: Analysis of Blazar Light Curves from *TESS*

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Abstract

Searching for low-dimensional chaos in time series is a rarely used, but very promising approach to reveal the systems' internal dynamics, which drives the observed temporal variability. In this work we use long and intermittent optical intensity time series from *TESS* satellite to search for signatures of low-dimensional attractors. We present briefly the commonly used for chaos searches *Correlation Integral* (*CI*) method and its application. This method is applied not only in astrophysics (including Solar physics), but also in the fields of meteorology, climate, geophysics, medicine, ecology, economy, etc. As an example, we apply the *CI* method to the *TESS* light curves of a relativistic jet-dominated active galactic nucleus - the blazar S5 0716+714. Even though our first results are inconclusive, such searches are promising and encouraging.

Keywords: blazar; variability; chaos

1. Introduction

The concept of *deterministic chaos* was introduced in 1960s with the realization that relatively simple systems of ordinary differential equations can have highly unpredictable (or *chaotic*) solutions. An example of a chaotic system is the *Lorenz system* (Equation 1), developed to model the atmospheric convection (Lorenz, 1963, see also Figure 1):

$$\left|\begin{aligned} \frac{dx}{dt} &= \sigma(y-x) \\ \frac{dy}{dt} &= x(\rho - z) - y \\ \frac{dz}{dt} &= xy - \beta z \end{aligned}\right. \tag{1}$$

Such solutions, called *strange attractors*, stay bound and never leave a sub-space of the phase space of (non-integer) dimension $d < N$, where N is the phase space dimension (the number of the variables/equations) of the system. The trajectories of a strange attractor evolve into a finite volume in the phase space, never returning to the same points. The divergence between two close trajectories increase exponentially in time and the long-term predictions are impossible. Consequently, each variable of a strange attractor system, represented as function of time, can mimic random behavior, yet still being a solution of a deterministic system.

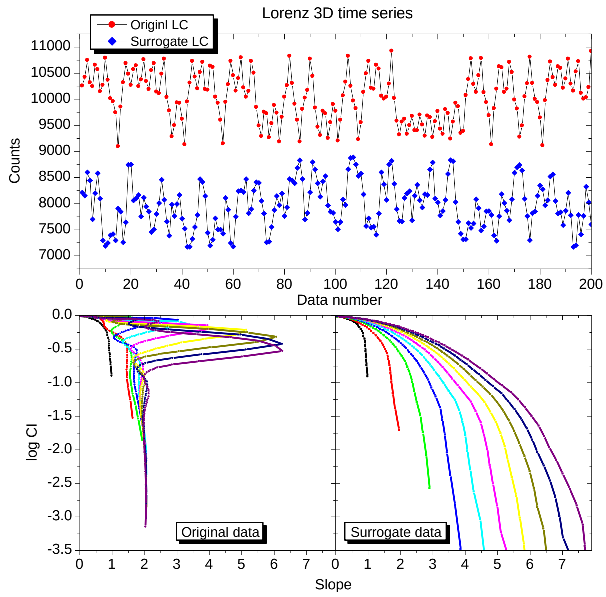

Figure 1. *Upper panel: Chaotic time evolution (the first 200 points) of the x-coordinate of the Lorenz system and the phase randomized surrogate (see the text), shifted vertically for clarity. Lower panels: Application of the CI method to the both curves (16000 points used). The original Lorenz curve (left) saturates at about* $Dc = 2.06$*, which is the correlation dimension of the Lorenz attractor, while the surrogate shows no sign of saturation and demonstrate behavior very similar to random noise (e.g. Bachev R. et al., 2015).*

Chaos theory applications include not only the field of astrophysics, including variable stars (Cannizzo et al., 1990; Bachev et al., 2011), Solar activity (Kurths & Herzel, 1987), active galactic nuclei (Lehto et al., 1993; Provenzale et al., 1994; Bachev R. et al., 2015; Bachev R. et al., 2018), X-ray binaries (Lochner et al., 1989; Misra et al., 2004), etc. but also many other disciplines as meteorology and climatology (wind directions, climate changes; Nicolis & Nicolis G., 1984; Fraedrich, 1986), geophysics (earthquakes intervals, Earth polarity reversals; Pavlos et al., 1994; Anufriev & Sokoloff, 1994), medicine (heart rhythm, epidemic outbreaks; Van der Heiden, 1991; Olsen & Schaffer, 1990), ecology (population size changes; May, 1986), economics (stock indices, exchange rates; Lorenz, 1989), etc.

In this work we will apply the *Correlation Integral* method (see the next section) to search for chaotic signatures in *TESS* light curves of a blazar (S5 0716+714). Blazars are active galactic nuclei, whose emission is almost entirely generated in a relativistic jet via non-thermal (synchrotron, Inverse-Compton) mechanisms and is additionally Doppler-boosted towards the observer (Urry & Padovani, 1995).

2. Correlation Integral method

The correlation integral method (Grassberger & Procaccia, 1983) is a relatively popular method in nonlinear dynamics. It is based on the construction of a new (empirical) phase space from the available N discrete observation points. The time series (the points of the light curve) is divided into strings of length d. Each string can be viewed as a d-dimensional vector (X_i) embedded in the d-dimensional empirical phase space. The number of pairs of vectors less than r apart as a function of r is calculated for different r and related to the total number of pairs (n_p) for that r. Thus, the correlation integral can be written as (Equation 2):

$$C_d(r) = \frac{1}{n_p} \sum_{i,j=1;\, j>i}^{N} \Theta\left(r - \left|X_i - X_j\right|\right), \quad (2)$$

where Θ is the Heaviside function. If the dimension of the attractor is D (Equation 3), then:

$$C_d(r) \propto \begin{cases} r^d, d < D \\ r^D, d > D \end{cases}, \quad (3)$$

Increasing d therefore leads to saturation at $d > D$, so the maximal *slope* of the CI can used to estimate the dimensionality of the attractor (Equation 4):

$$D_c = \lim_{r \to 0} \frac{d \ln C(r)}{d \ln r}, \quad (4)$$

where D_c is the correlation dimension of the attractor and may not be an integer. Knowing D_c allows determining the number of differential equations describing the dynamical system, N, i.e. the first integer greater than D_c. In this way, it becomes possible to draw conclusions about the physical processes leading to the variability.

The method also has some setbacks, for example, a quasi-periodic signal (with fractal dimension around one) accompanied by noise could be disguised as low-dimensional chaos of dimension 2 – 3. To prevent this possibility, it is necessary to test "surrogates" light curves obtained from the original curve by phase randomization (see e.g. Bachev R. et al., 2015). The surrogate curve preserves the frequency (Fourier) spectrum of the original, but destroys the appearance of deterministic chaos, if any. Thus, if manifestations of low-dimensional chaos are also observed in the surrogate curve, these manifestations will not be considered real (details in Bachev R. et al., 2015) and will be attributed to features in the frequency spectrum of the curve.

3. Data and results

For the purpose of this study we used light curve data from *TESS* orbital observatory (Ricker et al., 2015). *TESS* monitors continuously certain sectors from the sky for about 4 weeks with time resolution of about 2 (or 30 for some objects) min. Although primarily designed to search for extrasolar planets, *TESS* provides an additional and unique opportunity to study the variability of any type of object in the field. The resulting data cover the optical range in the red region (600 – 1000 nm) and are available as whole images or light curves (time series) for specific objects. The blazar S5 0716+714 was observed during several episodes (segments) and its light curves are used in our analysis as the objects is bright enough and rapidly variable. Each *TESS* segment contains typically a middle gap, large enough that each of the two parts to be treated separately. In addition, a few too deviating or missing points of the light curve had to be replaced via linear approximation between the adjacent points as the *CI* method works only for equidistant data series. Thus, each time series that we analysed consisted of about 8000 – 9000 equally spaced points, each measured with accuracy of < 1%. *TESS* segments 20, 26, 40 and 47, covering the field of S5 0716+714 were used for our analysis. Among the two flux calibration techniques (SAP and PDSCAP), we chose to use the SAP flux (details in Poore et al., 2024).

All tested light curves showed very similar behavior – a saturation around embedded dimension $d{\sim}1$ for both original and surrogate data. This means no detection of chaotic behaviour of the light curves, instead the light curves might be dominated by a (quasi) periodic signal (with $D = 1$ by definition). Further clues, supporting this possibility is that the surrogates demonstrate very similar behavior. In other words, the LC's of this blazar appear to be dominated by specific features of their Fourier spectra, not by deterministic chaos. Actually, similar results were obtained for another blazar, observed by *Kepler* - W2R 1926+42 (Bachev R. et al., 2015).

We show three examples (Figure 2 – 4) of *TESS* light curve (the original ones and the surrogates) of S5 0716+714 (parts from three different segments – 20, 26 and 40), all of which show strong variability and appear suitable for chaos searches. The time separation between data points is 2 min for all curves.

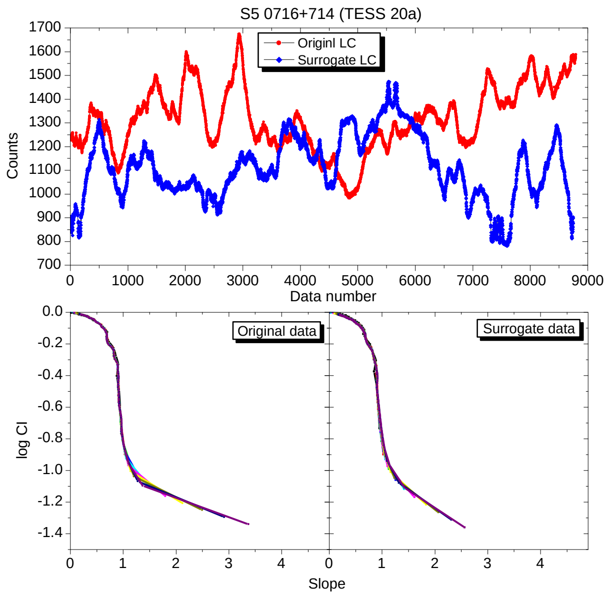

Figure 2. *An example of the application of the CI method to real light curve of a blazar (S5 0716+714, TESS, segment 20, first part, covering the period between 25.12.2019 and 06.01.2020) and its surrogate (see the text and Figure 1).*

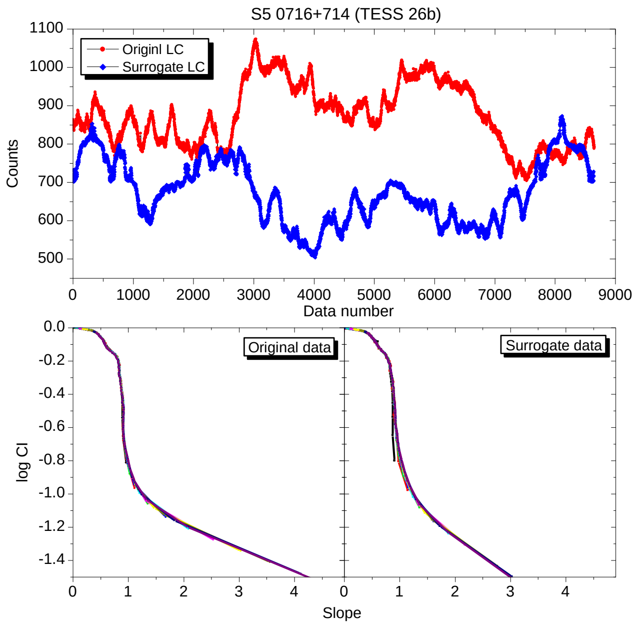

Figure 3. *An example of the application of the CI method to real light curve of a blazar (S5 0716+714, TESS, segment 26, second part, covering the period between 22.06.2020 and 04.07.2020) and its surrogate (see the text and Figure 1).*

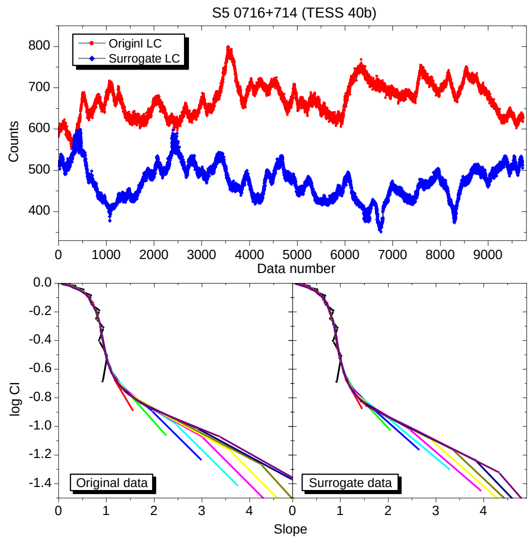

Figure 4*. An example of the application of the CI method to real light curve of a blazar (S5 0716+714, TESS, segment 40, second part, covering the period between 09.07.2021 and 23.07.2021) and its surrogate (see the text and Figure 1).*

4. Discussion

Finding low-dimensional chaos in a dynamical system (e.g. light curve) can be very important for the theoretical modelling, because it may limit the number of independent variables (or the number of equations) that governs the system entirely. In the case of blazars, such find may help to understand better the energy-producing mechanisms in a relativistic jet. The most often used is the so-called *single-zone* model to reproduce the observed blazar spectral-energy distribution (SED). In this model, the synchrotron radiation, which is responsible for the optical emission, is controlled by only a few parameters, such as electron density, electron energy distribution, magnetic field strength, Doppler factor, etc. Therefore, the single-zone models may actually produce low dimensional (say $D_c = 3 - 4$) chaotic behavior in blazar light curves.

One way to explain the apparently stochastic light curves, which would be consistent to a greater extent with our findings, is to invoke a large number of independent active zones (or emitting blobs) as an alternative to the single zone model. Another way to account for the observed variability is to invoke turbulence in the jet. In these models either turbulent flow passes through a standing shock (e.g. Marscher, 2014) leading to instant energy injection in the corresponding (shock crossing) plasma volume or the opposite: a shock wave passes through a turbulent medium (Marscher et al., 1992; Kirk et al., 1998).

5. Conclusions

We apply *chaos theory* methods to the *TESS* light curves of a relativistic jet-dominated active galactic nucleus – the blazar S5 0716+714. Although not able to detect the presence of low-dimensional chaotic behaviour in the light curves of this object, we find such studies important to restrain the emission mechanisms and the time evolution of blazars and other astrophysical objects. With the introduction of new or future ground- and space-based observational facilities, which can monitor the same objects repeatedly for very long periods of time, such studies will become even more feasible and perspective.

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